

## Maximising and Minimising

- Tip (Method)** 1. Write the quantity to be optimised as a function of *one* variable. (If two variables appear, eliminate one using the constraint given in the question.)
2. Differentiate, set the derivative to zero, solve.
  3. Justify maximum or minimum (second derivative, or gradient either side).
  4. Answer the actual question — often the value of the quantity, not of  $x$ .

**Example (Edexcel C2)**

The volume  $V \text{ cm}^3$  of a box of height  $x \text{ cm}$  is given by

$$V = 4x(5 - x)^2, \quad 0 < x < 5.$$

1. Find  $\frac{dV}{dx}$ .
2. Hence find the maximum volume of the box.
3. Use calculus to justify that your answer is a maximum.

**Example**

A farmer has 120 m of fencing for a rectangular pen against a long straight wall; the wall forms one side, fencing the other three. Find the dimensions giving the greatest area, and that area.

**Textbook Exercises:** SPS Course 6.1, Exercise 5 (optimisation questions)

## Optimisation with Constraints

### Example (Edexcel C2)

A solid glass cylinder has volume  $75\pi \text{ cm}^3$ . Polishing costs £2 per  $\text{cm}^2$  for the curved surface and £3 per  $\text{cm}^2$  for the circular top and base. The radius is  $r \text{ cm}$ .

1. Show that the cost of polishing is  $C = 6\pi r^2 + \frac{300\pi}{r}$ .
2. Use calculus to find the minimum cost, to the nearest pound.
3. Justify that your answer is a minimum.

**Example**

An open-topped box is made from a square sheet of card of side 24 cm by cutting a square of side  $x$  cm from each corner and folding up the sides.

1. Show that the volume of the box is  $V = x(24 - 2x)^2$ .
2. Find the value of  $x$  that maximises the volume, and the maximum volume.

**Exercise.** A closed cylindrical can must hold  $330 \text{ cm}^3$ . Find, in terms of  $\pi$ , the radius that minimises the surface area, and show that the optimal can has height equal to its diameter.

**Textbook Exercises:** SPS Course 6.1, Exercise 6 (Miscellaneous problems)